ETH Zürich, D-INFK
HS 2017, 11. December 2017

Prof. Ueli Maurer
Marta Mularczyk

Diskrete Mathematik

Exercise 13

13.1 Predicate logic with identity

We extend the syntax and the semantics of predicate logic to include the equality symbol "=" as follows:

Syntax: If t_1 and t_2 are terms, then $(t_1 = t_2)$ is a formula.

Semantics: If F is of the form $(t_1 = t_2)$ for terms t_1 and t_2, then $\mathcal{A}(F) = 1$ if and only if $\mathcal{A}(t_1) = \mathcal{A}(t_2)$.

a) ($\star\,\star$) Let $F := \forall x \forall y\ (x = y)$ and $G := \exists x \exists y\ \neg(x = y)$. Find necessary and sufficient conditions for a structure $\mathcal{A}$ to be a model for F and, respectively, for G. Justify your answer.

b) ($\star\,\star$) Find a formula H with identity, such that for structure $\mathcal{A}$ suitable for H, we have $\mathcal{A}(H) = 1 \iff |U^{\mathcal{A}}| \geq 3$.

13.2 Prenex form ($\star$)

Find a formula in prenex form, which is equivalent to F.

$$F := \forall z \exists y \big(P(x, g(y), z) \lor \neg \forall x Q(x)\big) \land \neg \forall z \exists x\ \neg R(f(x, z), z).$$

13.3 Logical consequence ($\star\,\star$)

Let F and G be any formulas of predicate logic. Which of the following statements are true?

a) $\models\ F\ \lor\ \exists y\ \neg F$

b) $\exists x\ F\ \land\ \exists y\ G\ \ \models\ \ F \land G$

c) $\{\forall x\ F, \forall y\ G\}\ \ \models\ \ F \land G$

13.4 The barber of Zürich ($\star$)

Use Theorem 6.11 to show that there does not exist in Zürich a barber who shaves all those and exactly those who do not shave themselves.

13.5 The Exercise 13 ($\star\star\star$)

Prove the following statement about people attending the Discrete Mathematics lecture:

> "There exists a person, such that if this person solves Exercise 13, then all people solve Exercise 13."

To this end, proceed in following steps:

a) Let U be the set of all people attending the Discrete Mathematics lecture. Let further P be the unary predicate, which is true if a given person solves Exercise 13. Using the predicate P, describe the above sentence as a formula F.

b) Show that F is a tautology (that is, show that it is true for any U and P).

c) Find a different (interesting) interpretation for F, which defines U and P.

No handing in.